For the free vibration analysis of the system in the figure we set $F_1(t) = F_2(t) = 0$ Further if the damping is disregarded $c_1 = c_2 = c_3 = 0$ and the equation of motion reduced to

$$m_1 \ddot{x}_1(t) + (k_1 + k_2) x_1(t) - k_2 x_2(t) = 0$$

$$m_2 \ddot{x}_2(t) - k_2 x_1(t) + (k_2 + k_3) x_2(t) = 0$$

Free vibration analysis of an undamped system

- Assume that it is possible to have harmonic motion of m_1 and m_2 at the same frequency ω and the same phase angle ϕ

$$m_1 \ddot{x}_1(t) + (k_1 + k_2) x_1(t) - k_2 x_2(t) = 0$$

$$m_2 \ddot{x}_2(t) - k_2 x_1(t) + (k_2 + k_3)(t) = 0$$

$$x_1(t) = X_1 \cos(\omega t + \phi)$$

$$x_2(t) = X_2 \cos(\omega t + \phi)$$

$$\left[\left\{ -m_1 \omega^2 + (k_1 + k_2) \right\} x_1 - k_2 x_2 \right] \cos(\omega t + \phi) = 0$$

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we shall consider the external Forces to be harmonic

$$F_j(t) = F_{j0} e^{i\omega t}, \quad j = 1, 2$$

$$x_j(t) = x_j e^{i\omega t} \quad j = 1, 2$$

We obtain

$$\begin{bmatrix} (-\omega^2 m_{11} + i\omega c_{11} + k_{11}) & (-\omega^2 m_{12} + i\omega c_{12} + k_{12}) \\ (-\omega^2 m_{12} + i\omega c_{12} + k_{12}) & (-\omega^2 m_{22} + i\omega c_{22} + k_{22}) \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

$$= \begin{Bmatrix} F_{10} \\ F_{20} \end{Bmatrix}$$

$$Z_{rs}(i\omega) = -\omega^2 m_{rs} + i\omega c_{rs} + K_{rs} \quad r, s = 1, 2$$

$$[Z(i\omega)] \vec{x} = \vec{F_0}$$

$$[Z(i\omega)] = \begin{bmatrix} Z_{11}(i\omega) & Z_{12}(i\omega) \\ Z_{12}(i\omega) & Z_{22}(i\omega) \end{bmatrix}$$

$$\vec{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \qquad \vec{F_0} = \begin{bmatrix} F_{10} \\ F_{20} \end{bmatrix}$$

The equation $[z(i\omega)]\vec{x} = \vec{F_0}$

$$x = [z(i\omega)]^{-1}\vec{F_0}$$

$$[z(i\omega)]^{-1} = \frac{1}{z_{11}(i\omega)z_{22}(i\omega) - z_{12}^2 i\omega}\begin{bmatrix} z_{22}(i\omega) & -z_{12}(i\omega) \\ -z_{12}(i\omega) & z_{11}(i\omega) \end{bmatrix}$$

$$x_1(i\omega) = \frac{z_{22}(i\omega)F_{10} - z_{12}(i\omega)F_{20}}{z_{11}(i\omega)z_{22}(i\omega) - z_{12}^2(i\omega)}$$

$$x_2(i\omega) = \frac{-z_{12}(i\omega)F_{10} + z_{11}(i\omega)F_{20}}{z_{11}(i\omega)z_{22}(i\omega) - z_{12}^2 i\omega}$$

By substituting these onto the below equation, the solution can be obtained

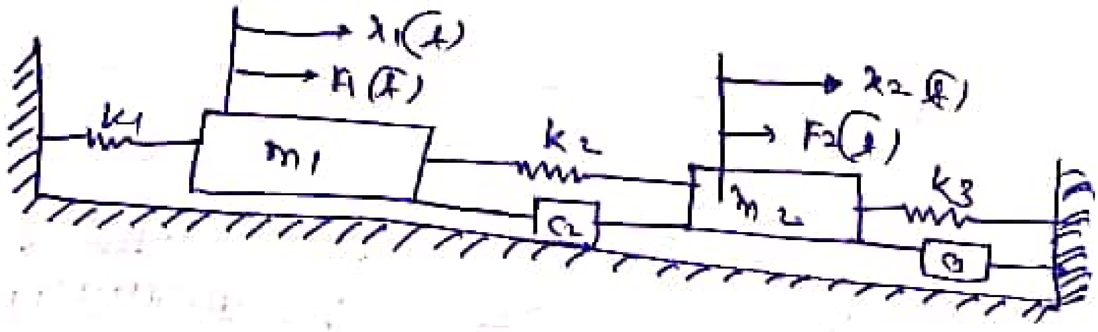
$$x_j(t) = x_j e^{i\omega t}$$

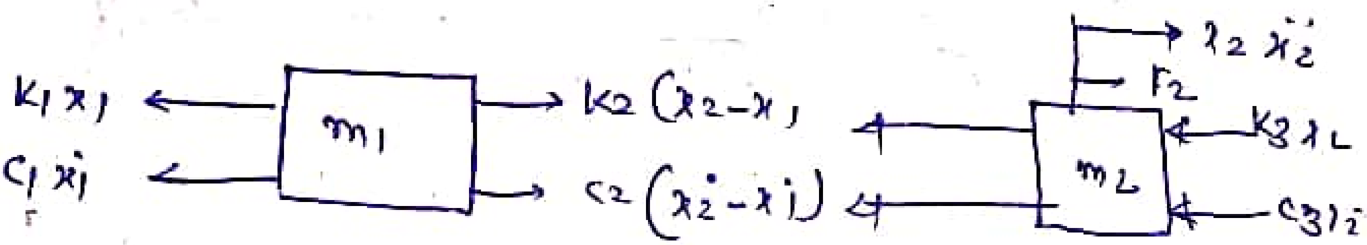
$$\boxed{j = 1, 2}$$

Equation of motion for forced vibration

Consider a viscously damped two degree of freedom spring mass system shown in fig.

The motion of the system is completely described by co-ordinate $x_1(t)$ and $x_2(t)$





$$m_1 \ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2 \dot{x}_2 + (k_1 + k_2)x_1 - k_2 x_2 = F_1$$

$$m_2 \ddot{x}_2 - c_2 \dot{x}_1 + (c_2 + c_3)\dot{x}_2 - k_2 x_1 + (k_2 + k_3)x_2 = F_2$$

Equation of motion for forced vibration

$$[m]\ddot{x}(t) + [c]\dot{\vec{x}}(t) + [k]\vec{x}(t) = \vec{F}(t)$$

$$m^2\omega^4 - 4km\omega^2 + 3k^2 = 0$$

$$\omega_1 = \left\{ \frac{4km - \left[16k^2m^2 - 12m^2k^2\right]^{1/2}}{2m^2} \right\}^{1/2} = \sqrt{\frac{k}{m}}$$

$$\omega_2 = \left\{ \frac{4km + \left[16k^2m^2 - 12m^2k^2\right]^{1/2}}{2m^2} \right\}^{1/2} = \sqrt{\frac{3k}{m}}$$

$$r_1 = \frac{(x_2)^{(1)}}{(x_1)^{(1)}} = \frac{-m_1\omega_1^2 + (k_1+k_2)}{k_2} = \frac{k_2}{-m_2\omega_1^2 + (k_2+k_3)}$$

$$r_2 = \frac{x_2^{(2)}}{x_1^{(2)}} = \frac{-m_1\omega_2^2 + (k_2+k_1)}{k_2} = \frac{k_2}{-m_2\omega_2^2 + (k_2+k_3)}$$

The amplitude ratio are given by

$$r_1 = \frac{(x_2)^{(1)}}{x_1^{(1)}} = \frac{-m\omega_1^2 + 2k}{k} = \frac{k}{-m\omega_1^2 + 2k} = 1$$

$$r_2 = \frac{x_2^{(2)}}{x_1^{(2)}} = \frac{-m\omega_2^2 + 2k}{k} = \frac{k}{-m\omega_2^2 + 2k} = -1$$

From $$x^{(1)}(t) = \begin{Bmatrix} x_1^{(1)}(t) \\ x_2^{(1)}(t) \end{Bmatrix} = \begin{Bmatrix} x_1^{(1)} \cos(\omega_1 t + \phi_1) \\ r_1 x_1^{(1)} \cos(\omega_1 t + \phi_1) \end{Bmatrix} = \text{1st m.}$$

$$x^{(2)}(t) = \begin{Bmatrix} x_1^{(2)}(t) \\ x_2^{(2)}(t) \end{Bmatrix} = \begin{Bmatrix} x_1^{(2)} \cos(\omega_2 t + \phi_2) \\ r_2 x_1^{2} \cos(\omega_2 t + \phi_2) \end{Bmatrix} = \text{2nd}$$

since the above equation must be satisfied for all values of time t the terms between brackets must be zero.

$$\{-m_1\omega^2 + (k_1+k_2)\}x_1 - k_2x_2 = 0$$

$$-k_2x_1 + [-m_2\omega^2 + (k_2+k_3)]x_2 = 0$$

$$\det\begin{bmatrix} \{-m_1\omega^2 + (k_1+k_2) & -k_2 \\ -k_2 & \{m_2\omega^2 + (k_2+k_3)\} \end{bmatrix} = 0$$

$$(m_1m_2)\omega^4 - \{(k_1+k_2)m_2 + (k_2+k_3)m_1\}\omega^2 + (k_1+k_2)$$

$$(k_2+k_3) - k_2^2] = 0$$

The above equation is called the frequency or characteristic equation.

$$\omega_1^2, \omega_2^2 = \frac{1}{2}\left\{\frac{(k_1+k_2)m_2 + (k_2+k_3)m_1}{m_1m_2}\right\}$$

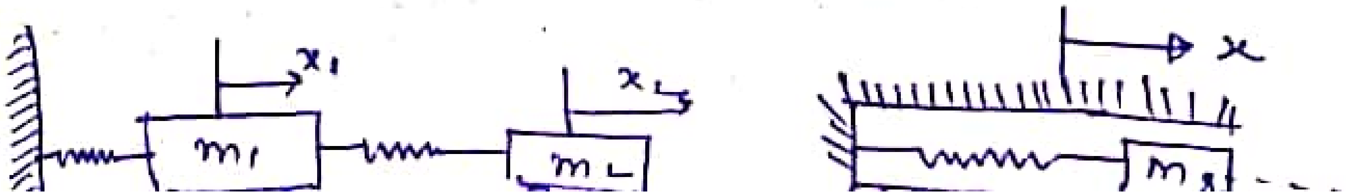
$$\pm \left[\left\{\frac{(k_1+k_2)m_2 + (k_2+k_3)m_1}{m_1m_2}\right\}^2 - 4\left[\left\{\frac{(k_1+k_2)(k_2+k_3) - k_2^2}{m_1m_2}\right\}\right]\right]^{1/2}$$

UNIT-3

Two degree OF Freedom

system that require two independent co-ordinates to describe their motion are called two degree of freedom system

NO OF DOF = NO of masses in the system × NO of possible types of motion of each mass



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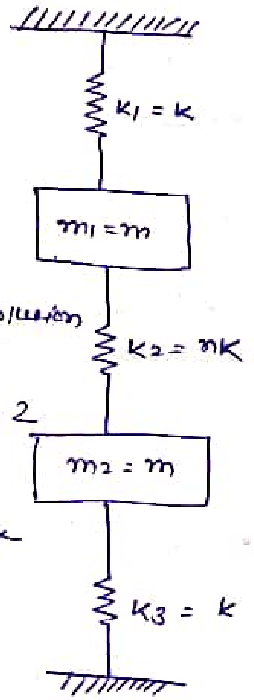
Q(1) Frequencies of mass - spring system

Find the natural frequency and mode shapes of a spring mass system, which is constrained to more in the vertical direction.



The equation of motion are given

$$m_1\ddot{x}_1 + 2kx_1 - kx_2 = 0$$

$$m\ddot{x}_2 - kx_1 + 2kx_2 = 0$$

By assuming harmonic solution

$$x_i(t) = X_i \cos(\omega t + \phi) \quad i = 1, 2$$

The frequency equation can be obtained by

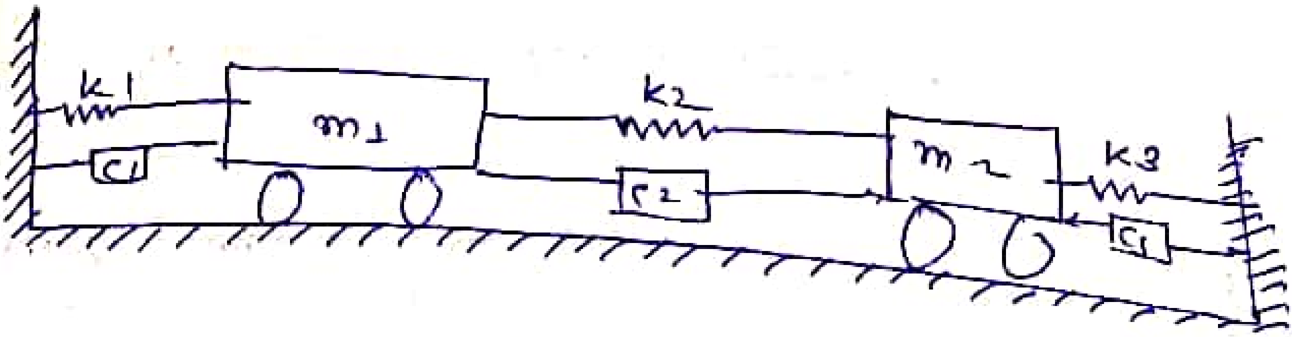
$$\begin{vmatrix} (-m\omega^2 + 2k) & (-k) \\ (-k) & (-m\omega^2 + 2k) \end{vmatrix} = 0$$

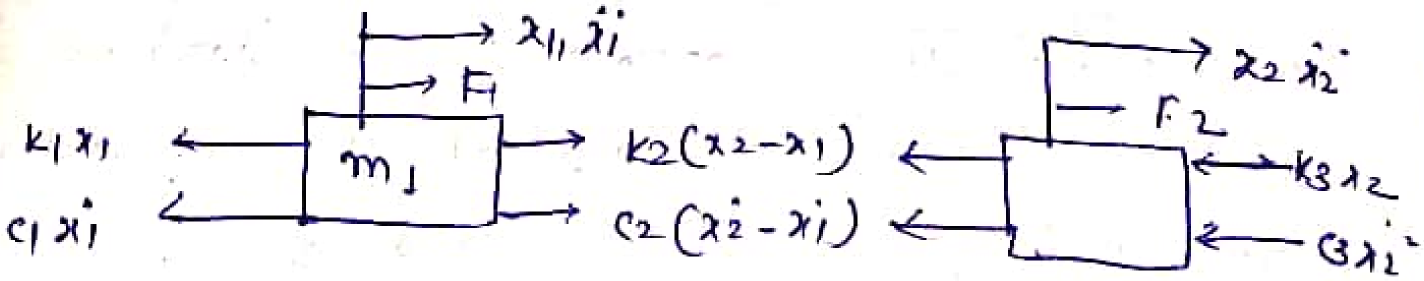
$$m^2\omega^4 - 4km\omega^2 + 3k^2 = 0$$

$$\begin{vmatrix} (-m\omega^2 + 2k) & (-k) \\ (-k) & (-m\omega^2 + 2k) \end{vmatrix} = 0$$

$$m = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$c = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{bmatrix}, \quad [k] = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix}$$





$$[m]^T = [m]$$

$$[c]^T = [c]$$

$$[k]^T = [k]$$

Free vibration analysis of an undamped system

$$x^{(1)}(t) = \begin{Bmatrix} X_1^{(1)} \cos\left(\sqrt{\frac{k}{m}}\, t + \phi_1\right) \\ X_1^{(1)} \cos\left(\sqrt{\frac{k}{m}}\, t + \phi_1\right) \end{Bmatrix}$$

$$x^{(2)}(t) = \begin{Bmatrix} X_1^{(2)} \cos \sqrt{\frac{3k}{m}}\, t + \phi_2 \\ - X_1^{(2)} \cos \sqrt{\frac{3k}{m}}\, t + \phi_2 \end{Bmatrix}$$

The motion of the system can be expressed as

$$\boxed{\begin{aligned} x_1(t) &= X_1^{(1)} \cos\left(\sqrt{\frac{k}{m}}\, t + \phi_1\right) + X^{(2)} \cos\left(\sqrt{\frac{3k}{m}}\, t + \phi_2\right) \\ x_2(t) &= X_1^{(1)} \cos\left(\sqrt{\frac{k}{m}}\, t + \phi_1\right) - X_1^{2} \cos\left(\sqrt{\frac{3k}{m}}\, t + \phi_2\right) \end{aligned}}$$

Forced vibration analysis

The equation of motion of a general two degree of freedom system under external forces

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}$$

$$+ \begin{bmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

→ modern accelerometer are the smallest MEMS consisting of a cantilever beam with proof mass. Accelerometer are avilable as two dimensional and three dimensional forms to measure velocity along with orientation. when the upper frequency range, high temperature range, and low packaged weight are required, piezoelectric accelerometer are the best choice.

Application

→ For inertial navigation system, highly sensitive accelerometer are used.

→ To detect and monitor vibrations in rotating machinery.

→ To display images in an upright position on screens of digital cameras.

→ For flight stabilization in drones.

→ Accelerometer are used to sense orientation co-ordinate acceleration, vibration shock.

→ used to detect the position of the device in laptops and mobiles.

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→ As ground motion occurs in three dimensional space axis or two horizontal axes displacement need to be recorded using three separate pendulum this is the only way that seismic activity can be comprehensively documented.

→ seismometer usually measure all three axes, ox for each direction in space. simple one axis seismometer usually only measure vertical motion, horizontal surface motion is ignored since it is not measured.

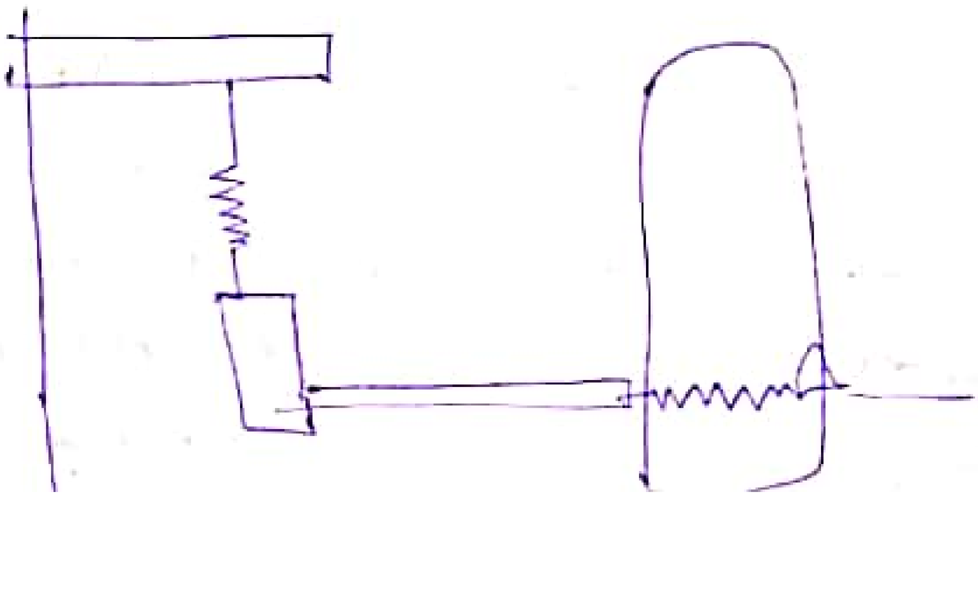
→ The sensitivity of a seismometer depends on the relationship between the seismic signal that you wish to record the various disturbances that interfere with this signal. this noise which can go as far as preventing measurement is simillar to the snow effect you can see on television screen.

UNIT - 4

* principle of seismometer

→ A seismometer is a device that is sensitive to vibrations. It works on the principle of a pendulum a heavy inert mass with a certain resistance to movement due to its weight is suspended from a frame by a spring that allows movement.

→ The energy from any seismic activity excites the "proof mass" as it is called by geophysicists making it vibrates.



(1) $\begin{cases} 1 & \text{if } -b<x<b \\ 0 & \text{otherwise} \end{cases}$	$\sqrt{\frac{2}{\pi}}\,\frac{\sin b\omega}{\omega}$
(2) $\begin{cases} 1 & \text{if } b<x<c \\ 0 & \text{otherwise} \end{cases}$	$\frac{e^{-ib\omega}-e^{-ic\omega}}{i\omega\sqrt{2\pi}}$
(3) $\frac{1}{x^2+a^2}\ (a>0)$	$\sqrt{\frac{\pi}{2}}\,\frac{e^{-a\lvert\omega\rvert}}{a}$
(4) $\begin{cases} x & \text{if } 0<x<b \\ 2x-b & \text{if } b<x<2b \\ 0 & \text{otherwise} \end{cases}$	$\frac{-1+2e^{ib\omega}-e^{-2ib\omega}}{\sqrt{2\pi\omega^2}}.$
(5) $\begin{cases} e^{-ax} & \text{if } x>0 \\ 0 & \text{otherwise} \end{cases}$	$\frac{1}{\sqrt{2\pi}\,(a+i\omega)}$
(6) $\begin{cases} e^{iax} & \text{if } -b<x<b \\ 0 & \text{otherwise} \end{cases}$	$\sqrt{\frac{2}{\pi}}\,\frac{\sin b(\omega-a)}{\omega-a}$
(7) $\begin{cases} e^{iax} & \text{if } -b<x<c \\ 0 & \text{otherwise} \end{cases}$	$\frac{e^{(a-i\omega)c}-e^{(a-i\omega)b}}{\sqrt{2\pi}\,(a-i\omega)}$
(8) $\begin{cases} e^{iax} & \text{if } b<x<c \\ 0 & \text{otherwise} \end{cases}$	$\frac{i}{2\pi}\,\frac{e^{ib(a-\omega)}-e^{ic(a-\omega)}}{a-\omega}$

$$= -\frac{2\cos nx}{n}\Big|_0^{\pi}$$

$$= -\frac{2[1-(-1)^n]}{n} = 0 \begin{cases} 0 & \text{if } n = \text{even} \\ & n = \text{odd} \end{cases}$$

$$\boxed{\text{Fourier series of } f = 1 + \sum_{m=0}^{\infty} \frac{4}{(2m+1)} \sin(2m+1)x}$$

Fourier transformation

The fourier transform is a magical mathematical tool. The fourier transform decompose any function into a sum of sinusoidal basis function. each of these basis function is a complex exponential of diffrent frequency.

$$\Big\{ \frac{1}{\quad} \; 9f \; -b \,.$$

Rotating plastic scanner probes

the Rotating plastic scanner probe section feature a variety of expandable plastic tip probes and stainless steel back shell probes, including the SUB series, SPO 5965, SPO 3564 series and standard series.

manual bolt hole probe

manual bolt hole probe coils are positioned at right angles to the shaft direction. these probe are rotated by hand to inspect holes with the fastners removed.

spot probe

spot probe are used for discovering flaws both on and below surface. their large coil diameter and low frequency operation are advantageous for scanning larger areas.

weld probe

designed to inspect ferrous magnetic

Eddy current based displacement probe

→ The eddy current type displacement sensors produce a high frequency magnetic field by applying a a high frequency current to the coil inside the sensor head.

→ The eddy current type displacement sensor measure the distance based on the change of the oscillation caused by this phenomenon.

→ there are different types of probe is used

(1) High voltage probe

(2) VD series high voltage probe

(3) Rotating plastic scanner probe

(4) manual Bolt hole probe

(5) spot probe

(6) weld probe

(7) Right angle surface probe

(8) Straight shaft surface probe

(9) Angle shaft surface probe

(10) Flexible shaft surface probe

principle of Accelerometer

→ The basic underlying working principle of an accelerometer is such as a dumped mass on a spring. when acceleration is experienced by this device, the mass get displaced till the spring can easily more the mass with the same rate equal to the acceleration it sensed. Then this displacement value is used to measure the give the acceleration.

→ Accelerometer are avilable as digital devices and analog devices. accelerometers are designed using diffrent methods. piezo electric, piezo resistive and capacitive component are generally used to convert the mechanical motion caused in accelerometer into an electrical signal.

→ piezoelectric accelerometer are made up of single crystal. these use the piezoelectric effect to measure the acceleration. When applied to stress these crystals generate a voltage which is interpreted to determine the velocity and orienta

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)\,dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx)\,dx$$

<u>or</u>

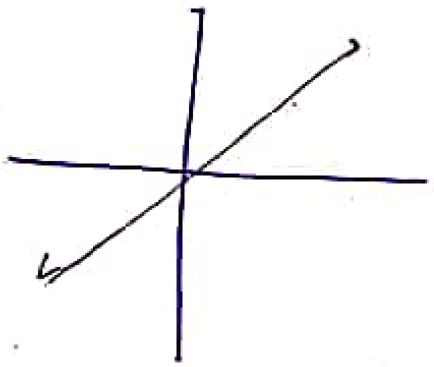
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

$$a_0 = \frac{1}{\pi} \int_{0}^{2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \cos(nx)\,dx$$

$$b_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \sin(nx)\,dx$$

<u>Example</u> (1) $f : [-\pi, \pi] \to R : x \longrightarrow x$



Here $f(x) = x$

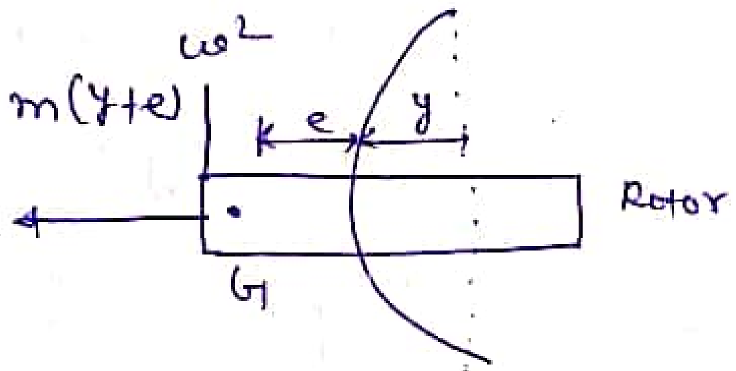
a odd function

$a_n = 0$

Right angle surface probes

90° tip stainless steel shaft. designed for general surface crack detection, these probes are available in a variety of lengths and with various coil configuration drops and connector options.

* Critical speed of simple shaft



Critical or whirling or whipping speed is the speed at which the shaft tends to vibrate violently in transverse direction

→ The eccentricity of C.G of the rotating masses from the axis of rotation of the shaft

→ Diameter of the disc

→ span(length) of the shaft

→ type of supports connection at the its end

$$b_n = \frac{1}{\pi}\int_{-\pi} x \sin nx\, dx = \frac{2}{\pi}\int_0 x \sin nx\, dx$$

$$= \frac{2}{\pi}\left\{\left[-x\frac{\cos nx}{n}\right]_0^{\pi} + \frac{1}{n}\int_0^1 1.\cos nx\, dx\right.$$

$$= \frac{2}{\pi}\left[-\pi\frac{\cos n\pi}{n} + \underbrace{\frac{\sin nx}{n^2}\Big|_0^{\pi}}_{0}\right]$$

$$= -\frac{2(-1)^n}{n}$$

$$\boxed{\text{Fourier series} = \sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n}\sin nx}$$

Q (2)

$$f:[-\pi,\pi]\to R : x \to \begin{cases} 0 & \text{if } x\in[-\pi,0] \\ 2 & \text{if } x\in(0,\pi] \end{cases}$$

Here F is Neither even nor odd

$$g(x) = f(x) - 1$$

is odd

$$b_n = 2\int_0^{\pi} 1.\sin nx\, dx$$

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Bending critical speed

The synchronous whirl frequency increases with the rotational speed linearly and can be represented 1X rev excitation frequency whenever this excitation line intersects the natural freq critical speed occur.

Fourier series

The Fourier series of the function $f(x)$ is given by. $f(x) = a_0 + \sum_{n=1}^{\infty} \{a_n \cos nx + b_n \sin nx\}$

where the Fourier coefficient a_0, a_n and b_n are defined by integral

→ Fourier series are used to resolve the signal into an infinite sum of sine and cosine terms. basically Fourier series is used to represent a periodic signal in terms of cosine and sine wave.

→ A Fourier series is an expansion of a periodic function. In terms of an infinite sum of sines and cosines.

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{+T_0/2} f(t)\,dt$$

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\cos(2\pi n t/T_0)\,dt$$

$$b_n = \frac{2}{T_0}\int_{-T_0/2}^{+T_0/2} f(t)\sin(2\pi n t/T_0)\,dt$$

$$\int_{-\pi}^{\pi} \sin(nx)\,d\cos(mx)\,dx = 0$$

$$\int_{-\pi}^{\pi} \sin(nx)\sin(mx)\,dx = \begin{cases} 0 & m \neq n \\ \pi & m = n \end{cases}$$

$$\int_{-\pi}^{\pi} \cos(nx)\cos(mx)\,dx = \begin{cases} 0 & m \neq n \\ \pi & m = n \end{cases}$$

For equilibrium

$$ky = m(y+e)\omega^2$$

$$\boxed{y = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}}$$

$$\boxed{\omega_c = \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{g}{\delta}}}$$

Dynamic Force on the bearing

$$\boxed{ky = m\omega_n^2 y}$$

Bending critical speed

Hence dynamic magnifier and phase angle

$$\boxed{H(\omega) = \frac{R}{a} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)}$$

For an undamped rotor resonance occurs

$$\omega = \omega_n$$

$$\varphi = 90^\circ$$

$$m\omega^2 a = c\omega R$$

$$\frac{R}{a} = \frac{m\omega}{c} = \frac{1}{2\xi}$$

$$\boxed{R_{max} = \frac{a}{2\xi}}$$

$$\boxed{R_G = a\sqrt{\frac{1+2\xi r}{(1+r)^2 + (2\xi r)^2}}}$$

bearing